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# Book Review: What Is Mathematics, Really? by Reuben Hersh

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*What Is Mathematics, Really?* Reuben Hersh. Oxford University Press.

The mad oddball maverick strikes again! This is a great book. What, you may ask, makes a book great? Here are my criteria:
1) I learn a great deal from it.
2) The book gives me new insights that change or reinforce previous viewpoints (prejudices?).
3) I agree with almost everything the author says.
4) It stimulates thinking.
5) It is written with clarity and with conviction.
In my opinion, Reuben satisfies all these in this book. What more could one ask for, except maybe a chocolate donut? Criteria number five is one which is not often met in books about mathematics and in mathematical writing in general. Recently reading two forwards to a book, one written by Reuben Hersh and one by another writer, reminded me of the great range in this quality and its importance in making reading pleasurable. Reuben's was far superior in this respect; it was like night and day. One wonders, sometimes, if mathematicians equate obscurity with scholarliness. Reuben has a pleasant, conversational style which is very refreshing in mathematical writing.

Is this what mathematics is—really? Well it's much closer than what we've seen before. It is a giant step in that direction; it's probably within an epsilon distance, even. I have always felt that "philosophical" concern was much like contemplating our mathematical navel. Also, I have often used Albert Einstein's quotation, "Is not all of philosophy as if written in honey? It looks wonderful when one contemplates it, but when one looks again, it is all gone. Only mush remains." You can imagine my consternation when I found out that this whole book was about the philosophy of mathematics, and I had paid good money for it. Is that what mathematics really is? Come on! Well, it turns out that it is, and it was well worth the money. Reuben builds a convincing case for this. Whether or not we articulate it or even think about it

as such, we all have a philosophy of mathematics that guides us in our teaching and mathematical work. He moves the philosophy of mathematics away from the realm of navel contemplation.

An important part of the book is a thorough discussion of the sundry philosophies of mathematics. The history of these philosophies is important in understanding the evolution of mathematics, but categorizing mathematicians, i.e., labeling them as belonging to one of these categories, bothers me. Labeling, or categorizing individuals, is a distancing phenomenon. It tends to be divisive. Actually, I think that is a part of the humanistic philosophy, that we must be cognizant of all the philosophies of mathematics and their contributions to mathematics and our culture in general. Reuben very convincingly argues that all the different philosophies, except the humanistic one, fail to satisfy the criteria of THE PHILOSOPHY OF MATHEMATICS that he puts forth, but he does recognize their importance in the evolution of mathematical thought.

If, many years ago, someone would have asked me what kind of mathematician I was, formalist, Platonist, intuitionist, foundationalist, I would have answered yes. There are certain ideas in each of these that I subscribe to, and I think that is consistent with what Reuben is preaching. Actually, I guess I have always been a humanist as described here. However, I didn't know it and didn't know there was such a thing until I attended an open forum on Mathematics as a Humanistic Discipline at the International Congress of Mathematicians at Berkeley in 1986. Although as a student, I really ate up the courses that were heavy with formalism and foundations; I had great admiration for the writings of Raymond Wilder, Morris Kline, Edward Kasner, James Newman, Edna Kraemer, Lillian Lieber, and the like. So, at heart, I guess I was and am essentially a humanist. Reuben describes humanistic mathematics as social-cultural-historical. Doesn't cultural say it all? Perhaps that is not obvious

to all. The book has lots of advice for philosophers of mathematics, such as "Give up the illusion of mathematical precision, aim for insight, enlightenment." This is very good advice for all who work with mathematics. He also has some good advice for teaching mathematics, such as how to use proof in the classroom, not to convince so much as to help explain and as an aid to understanding. Maybe I'm reading something into this, but I think his discussion seems to support my criticism of most "liberal arts" math courses and textbooks. They seem too concerned with "doing" mathamatics rather than aiming for understanding the thought processes in the evolution of mathematics. There is a great deal of excellent advice in this book for anyone involved in teaching mathematics, which is somewhat surprising since it comes from someone who used to teach the "wrong stuff."

There is a nice discussion on the meaning of numbers which exemplifies the fuzzy thinking that can result from using a simple term in more than one way without clarifying how it is used. For example, the number 2 as used as an adjective, as in 2 bananas and as a noun, as in a number, the consequent of 1 a la Peano; it has a multiplicative inverse; it is an integer, a rational number, a real number, a complex number. Students learn numbers as adjectives, and then we start using them as nouns without discussing this difference. This leads to confusion and fuzzy thinking. All this discussion of nouns and adjectives, you would think that this author is some kind of English major. However, mathematics is primarily a language, and it would behoove all of us to keep that in mind—always. There is much discussion (argumentation) in the philosophy of mathematics that is merely semantics, lack of clarity, fuzzy thinking. There is much ado about whether the things of mathematics are "objects," are "real", whether mathematics is (are?) "invented" or "discovered." Those discussions sound so erudite, but I think that is the sort of thing Einstein was alluding to. Perhaps I am being naïve, but who cares about all those types of discussions? The things of mathematics are ideas. Ideas are real, and what is important is that they are useful and important and have been and will be essential in the development of our culture. There is much discussion of philosophy of mathematics in this book that is just that, the importance of mathematics in the development of the culture.

There is an excellent discussion on the myths in mathematics. You should read this for yourselves, and we all should keep them in mind. We too often fall victim of these myths, and that leads us to less than adequate presentations. The author states,"the standard exposition purges mathematics of the personal, the controversial, the tentative, leaving little trace of humanity in the creator or the consumer." Of course, he has fallen victim to a myth himself in assuming there is a "standard exposition." This, however, does not detract from the importance of what he is saying. Expositions in mathematics far too often are as he has stated. Let's work on that, shall we?

In my opinion, in attempting to discuss what mathematics is in terms of humanistic views, our language fails us, although Reuben does it much more elegantly than anyone else. It seems that couching the discussion in philosophical, technical, scholarly language distances us from what we really believe. The essence of mathematics is its open-mindedness. Mathematics is different things; the key is to be open-minded and tolerant of the views of others.

The author makes a very cogent statement, "Mathematics is a lawful, comprehensible evolution from a basic core. It develops in response to internal strain (here a definition would help) and external pressure." Raymond Wilder referred to these as "hereditary and environmental stress." Perhaps this would suffice as the definition that the author says would help. If not, and even if so, a reading of Wilder's "Evolution of Mathematical Concepts" would be of interest.

Reuben's use of the blind dudes and the elephant as a metaphor for the sundry views of mathematics is nicely done. It seems to me I have seen this before, but Reuben does it much better than anyone.

I think I have earned the right to be a curmudgeon so I am going to voice a wee criticism. I think Reuben has overdone it a bit relating mathematical philosophies with political philosophies. It seems to me like belaboring the obvious, but he made his point, and maybe others are not so hard to convince as I.

Did I say this is a great book? I highly recommend it. I believe it should be required reading along with Wilder's "Evolution of Mathematical Concepts" and Morris Kline's "Mathematics in Western Culture" for any student in a mathematics program.