

Winter 1-1-2004

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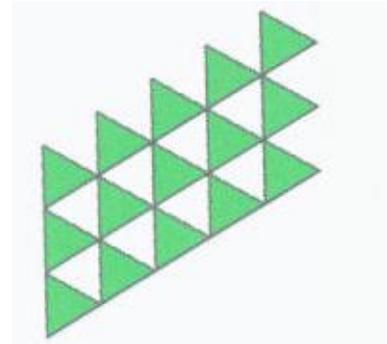
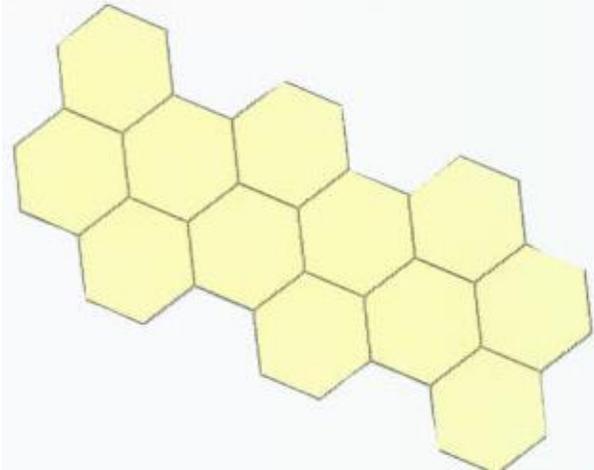
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Recommended Citation

Glover, Martin (2004) "Tesselland: A Mathematical Oddment," *Humanistic Mathematics Network Journal*: Iss. 27, Article 6.
Available at: <http://scholarship.claremont.edu/hmnj/vol1/iss27/6>

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Tesselland
A Mathematical Oddment

Martin Glover
Milligan College

I had just finished Mr. Abbott's *Flatland*^{*}, reclined in my favorite easy chair for a peaceful afternoon nap and, still in a state of spatial titillation courtesy of Mr. Abbott, I fell into what is best described as a trance. It was as if I had become an observer out of time, out of space, yet aware of all, and drawn irresistibly toward what appeared most certainly to be a lecture; a lecture delivered by a disheveled, bearded and aging professor to a modest collection of equally disheveled but earnestly attentive pupils sparsely populating a huge and ornate lecture hall. His monologue echoed throughout the cavernous confines for lack of soft, sound-absorbing bodies and continued in this fashion.

“As you and I had no hand in our own creation, and being confounded at every turn to explain the curious cosmic how's, when's, where's, and why's of consciousness, a.k.a. the meaning of life, a Tessellate is equally at loss when considering her present state; that state, being confined in two dimensions, adjoined at every side by another replica of herself or by a not-so-dissimilar figure separating her from an exact likeness in close proximity.^{**}

“Her added disadvantage is one of corporeal identity. While we may struggle with quandaries of the soul, we do so as discrete physical entities. She, on the other hand, shares even that which makes her distinct with as many others as would adjoin her. Where her disjoint ends and her self begins, she can only scarcely guess. Fortunately, 'nature' has taken that into consideration and she expends precious little mental resource in unraveling this dilemma. This I am confident you will discover. Now back to our common predicament.

“She neither recalls her beginning of days nor has barely the vaguest of notions concerning her future. She occupies the moment. For that is all that even remotely explains itself, but

**Flatland* by Edwin Abbott is a novel about a two dimensional world. Its inhabitants are regular polygons, i.e. many-sided figures with congruent (equal) sides and congruent angles. The story is told from the viewpoint of one of its citizens, A. Square.

**A figure or group of figures that fit together in a plane of construction without overlapping or leaving gaps is called a tessellation. it then, cruelly, instantly, plunges into the past, replaced by another 'now'. The moment, relegated to a memory, becomes her only source of self-definition. It is her existence. She is propelled into her future by the momentum of her past. So there you have it. Contemplate her plight. Consider her Source. Judge her fate and report your findings; dutifully researched and properly footnoted at our next session. Remember – that is four weeks from today, seeing that I will be indisposed during our customary meeting period, attending to a pressing matter that will occupy all of my energy and mental faculty. Take nothing lightly and disturb me only during posted office hours, and even then only in the case of dire straits pertaining to your task.”

With that the venerable old fellow exited the hall as his slightly nonplussed students exchanged glances. Some looked confused, others bemused. Slowly, lazily, they gathered their bags and wraps, leaving randomly as students are prone to do. Three tarried. One, turned askew in his seat, his left leg hanging over the seat's back and right leg over the adjacent armrest, his back resting against the opposite adjacent armrest, was engaged in following the conversation between his two cronies seated behind.

“No Julia. You don't get it. It's allegory. All a-l-l-e-g-o-r-y. And bad allegory at that. Old Blanchard has finally lost his last tenuous hold on reality. This assignment might as well be entitled *Locke, Leibniz, and Hume: A Take-Home Exam*. What was it he said...ummm..’ She neither recalls her beginning of days nor has barely the vaguest of notions concerning her future.....the moment is all that even remotely explains itself,...relegated to a memory, becomes her only source of self-definition...propelled into her future by the momentum of her past. ...blah, Blah, BLAH!?’ Come on: Hume – our sense of identity is not continuous; Leibniz - apperception, the inner state's self awareness; Locke – intuitive knowledge of self, demonstrative knowledge of God, whoever She is, and sensitive knowledge of...well, surely you see the ruse!”

“How can you always be so certain about any and everything? He's not being coy this time. I can't explain it yet, but I'm sure he's given us a seriously original assignment.”

“Poor Julia, you should have been named Patsy. I'd bet...”

“I think Julia is right Cayley.”

That statement seemed to take Cayley off guard. Appearing taken aback, even slightly offended judging from her raised eyebrows and faint pout, she fired a glance down at her reclining friend. Julia appeared surprised as well.

“Well, that's a first. Ian takes simpleminded Julia's side against the erudite and enlightened Anne Margaret Cayley. Should I be honored or alarmed?”

Ian sat upright, giving a disarming smile to both young women and continued with a bit more energy and enthusiasm, carefully signifying by his tone that he was in no way disputing Cayley's superior intuition.

"It's true that Blanchard is monotonously notorious for deflecting existential philosophy onto unsuspecting surrogates. The poor old Prof still believes each time that he has pulled one over on us. And he's always quite smug when we, acting the foil, appear startled and enlightened as we discover, unwitting pawns that we are with our feeble notions, we have been the object in disguise of his cleverly conceived lesson."

"Ohhh, and I want to just stand up and look him in the eyes over those warped reading lenses resting on the tip of his pointy nose and say, 'Blanchard, you senile academe! We've known it all along. And the time before, and the time before that, and...'"

"CAY! Please go on Ian."

Giving Cayley a wink, at which she harrumphed, shrugged her shoulders and eased back into her seat, Ian proceeded.

"Remember Prof's dissertation?"

Julia and Cayley exchanged puzzled looks. Ian grinned.

"C'mon Cay. You're the one who read it the day you learned he was going to be the lecturer in first year Philosophy and Religion. What struck you as odd, even back then; so odd, in fact, that you mentioned it to Professor Glogov, the Math Chair?"

Cayley thought hard and then it dawned, that almost marginal comment that seemed so out of place it was provocative.

"I know what you mean, Ian. It was those two or so comments about space, time, dimensions, and consciousness...or something like that!?"

"Exactly. None of those themes are unusual for a doctor of philosophy dissertation abstracting Kant's Quantity, Quality, Relation, and Modality. What was it that made his treatment so peculiar?"

Clearly thinking quite hard, as if she were rising to a challenge, it became clear.

"I remember now", Cayley exclaimed with an almost visible light beaming to life above her tightly curled hair. "I remember it exactly!"

Ian and Julia rolled their eyes knowingly. Once Anne Margaret Cayley ingested information it was destined to remain forever on file, ready to be called up on demand. Cayley's finely tuned synapses rarely, if ever, misfired.

“Plato, borrowing and enhancing the Pythagorean philosophy, understood the essence but not the purchase of number. Space, time, dimension, and consciousness are not disparate. They are one, and God’s Equation shows it to be so.”

With a look of triumph, Cayley beamed with satisfaction at her friends, but before she could duly celebrate, her continence sobered.

“But I still don’t get it, Ian. Why does this imply old Prof isn’t up to his old tricks?”

“I’m a little foggy on the connection too, Ian. I’m glad you’ve taken up my cause in principle but what are you thinking?”

“First ladies, let’s stop by the pub for a little refreshment; then off for a bit of research. I’ll explain as we walk, but I believe we may be embarking on an adventure.”

“And by the way,” Ian turned, glancing toward Julia, “I think Patsy is a quite lovely name.”

Almost certainly, a slight blush rose on Julia’s cheeks as they turned to exit the lecture hall. Cayley definitely rolled her eyes and grimaced.

At that I awoke from my quasi-dream state. Every word and gesture, even the scent of the lecture hall and students clothes lingered vividly in my mind. When I related the matter to my wife, she smiled politely, saying, “That’s nice, dear,” and reminded me the rubbish was near overflowing the trash bin and needed my assigned attention. To this I came crashing back to reality and would have passed off my excursion to Professor Blanchard’s lecture as a pleasant, though bit unsettling experience, had it not been for what happened that evening. At the end of the day, the demands of which having pressed from my mind the events earlier visited, I retired for the evening after consuming my traditional bedtime brandy toddy. Just as before, when I stretched prone and relaxed for sleep, the same trance-like state possessed me. Immediately, and without recourse, I was propelled into a narrow passage, surrounded on either side by walls with a staggered, uneven paneling. It was such an odd confinement yet common. Then I heard voices, familiar voices. The three friends from the lecture hall were in serious and agitated conversation amplified over the ruffling of papers and turning of pages. Now it was clearer. I was in a library, presumably the library of the same university at which the lecture had take place earlier in the day. The architecture was of similar style to that of the lecture hall and the students, now visible milling through the aisles, had the same appearance as Dr. Blanchard’s. Floating above the stacks, like an otherworldly and invisible eavesdropper, I peered down upon the trio. Their discourse continued.

“OK, Ian. You’ve built an interesting argument, but for my and Julia’s sake”, Julia’s face shone the same perplexed look as before leaving for the pub, “please lay out your logic one more time.”

Ian drew a deep breath, sat up on the edge of his chair, gripped the sides of his seat and began.

“Mind you, this would not likely hold up in a court of law, but it is contextually sensible. Here goes. First, men of genius throughout the ages...”

“Ahem!”

“Sorry Cayley, men and women of genius throughout the ages, and in particular The Enlightenment, were generalists; experts in several fields. Descarte was a philosopher and mathematician, Newton was a theologian, alchemist, mathematician, and pioneered optics. The list goes on and on. No one specialized. Prof is precisely that kind of Renaissance man. Take note of this sampling of his publications:

Religion Without God
God Without Religion
The Philosophy of Nature
Crystalline Structure and Evolution
Dynamical Systems and Consciousness

and in particular these last three,” which lay open before them.

Metaphysical Mathematics
The Creator’s Dilemma
On Structure, Substance, and Soul.

Cayley leaned over to Julia, still fixed on Ian, and loudly whispered “I think Shelley had Prof in mind when she penned *Frankenstein*.” The three laughed. Ian caught his breath and continued.

“Now, moving forward on the assumption that Blanchard’s cryptic dissertation remarks are not figurative, we can only conclude he meant them literally. In short, as outrageous as it sounds, old Prof thinks he’s found the secret of creation, at least on some level. ‘God’s Equation’ is either a fundamental mathematical formulization for life that he already had or was on the verge of obtaining. It’s been decades since he penned those lines and I’m sure he’s had time to refine whatever process to which he was referring to the point that he has attempted to create some rudimentary form of life, possibly succeeding. Here, see what he writes in *Metaphysical Mathematics*.”

“*Contemporary mathematics has advanced to the stage where the gap between the ethereal and animal can be explained in precise terms. Not only can it be explained, it is closed. Nothing short of an analytical model lies within our grasp. Within the next decade we will see **Quod Erat Demonstrandum**.*”***

“And these two excerpts, the first from *On Structure, Substance, and Soul* followed practically immediately in publication from *The Creator’s Dilemma*.”

“Energy, as Mr. Einstein has recently intoned, is the crux. It unifies but it is as elusive and enigmatic as it is apparent. To channel it will mean to enliven. Literally! Inspiration, find me. Oh tarry not, sweet vision.”

“What then for the architect but to assume responsibility? We are fearfully made, fearfully we make.”

“It appears the scientific community passed these off as the ravings of a passionate, but misguided would be philosopher-scientist. Remember Glogov’s dismissal of Cayley’s question about Prof’s dissertation? He just smiled, told us Prof was a hopeless romanticist and should be taken with a ‘grain of salt’. The old guy is well liked among his colleagues so they’ve humored him but never taken him seriously. It’s a leap to believe he’s succeeded but, and granted this is a ‘but’ of literally biblical proportions, what if he has?”

The three sat silent for a moment. The Cayley, shaking her head as if clearing the cobwebs from a dazing blow, spoke up.

“Even if what you say is true Ian, and I’m at best skeptical, what is the purpose of his assignment?”

“I don’t know”, he said frankly and slumped in his chair.

As if Ian’s reaction triggered the opposite effect, Julia sprang up in her seat, her eyes bright with anticipation and excitement.

“Well, here’s a second leap in logic. I’ll answer that Cay. I think he’s in a fix and needs our help.”

“That is a leap!” Cayley shot back. “Sure Julia, he needs our assistance when he could go for help among his legion of professional associations, all of whom are infinitely more qualified than us.”

“Not infinitely Cay. Remember a few days prior to his last lecture he told us he had been unfortunate enough to watch the cynical peers of his youth become...”

“...the materialist colleagues of his present. Yeah. I remember. So what?”

Ian, seeing the connection, revived and chimed in.

“Because he dare not reveal his secret to anyone with ambition. Get it Cay? He doesn’t trust them and he’s hoping one from among our ranks will prove worthy.”

“Then just what kind of fix do you suspect he’s in my two hypothesizers?”

Ian and Julia paused, looked one another in the eye and in perfect unison turned to Cayley, saying,

***Historically, the initials Q.E.D indicate the conclusion of a mathematical proof. This abbreviates the Latin, Quod Erat Demonstrandum, which means “That Which Was To Be Demonstrated.”

“The Creator’s Dilemma!”

Cayley relaxed in her chair as if digesting a great meal. She put her head down into her hands. Ian and Julia appeared to prepare for a verbal onslaught but instead, after a moment, Cayley raised her head from her hands and with a most pleasant expression on her face, grinned at the two of them, nodding her approval. The skeptic was now a convert.

“Ian, your interesting argument has just become compelling. So what next?”

Julia leaned over the table and opened the dictionary to the t’s. The top of the page six hundred and fifty three heralded its contents, **terror/thank**. She pointed to a spot near the middle and their eyes converged on a definition.

tes-sel-ate (tes’ə lā t) *vt.* **-lated, -lating** [< L *tessella*, little square stone] to lay out in a mosaic pattern of small, square blocks. (Contemporary usage allows the mosaic pattern to consist of any collection of figures, most commonly geometric or alterations thereof, provided no overlaps or gaps exist.)

“This seems a good place to start.”

Sleep ensued. How late into the night the vision lasted I cannot tell, but I awoke the next morning refreshed, the library dialogue still clearly etched in my mind. While a common reaction might have been horror or fear of impending insanity, I felt elation and anticipation. It seemed I was caught in the wake of some mystical tide whose destined shore I longed to see. However, two weeks passed without further visitation with the students and their professor. Just as I was willing to disappointedly assign the episodes to one of those curiosities of life, my regret was truncated. It happened as on the fortnight previous after my evening toddy. This time I found myself in a hallway lined with offices, a janitor at the far end, mop and pail in tow. He was watchfully intent on three young people standing at one of the doors in the otherwise empty corridor. Ian was knocking repeatedly on the door and as I drew closer I could see painted on the opaque pane comprising the door’s upper half: Dr. Roosevelt Blanchard – Professor of Philosophy – Department Chair. Julia looked around uneasily as crashing noises came from within the office. The janitor began approaching the trio. Cayley wedged between Ian and the door.

“Dr. Blanchard. Please open the door. We know you’re in there. This is Anne Margaret Cayley with Ian Wainwright and Julia Spenser. We need to speak with you about the assignment you gave our class. It’s extremely important. We know this is not your posted hour but we’ve waited all day. Please sir, see us.”

Blanchard threw the door open with such force it startled the students. His eyes were wide and darting, like prey in search of escape from an imminent predator. He turned his gaze on the three, grabbed Cayley by the sleeve and pulled her inside.

“Quick. You two come in.”

Involuntarily, Ian and Julia obeyed. The professor’s clothes were wrinkled and tattered, his visage bedraggled.

“Wainwright! I was confident you and your friends would be the first. I have much to tell you but we have to leave my office. Out this way.”

Blanchard pointed toward a bookshelf in the corner of his office, an unlikely looking exit. The students paused but Blanchard herded them promptly toward the corner, bent to the lowest shelf and pulled the first book from its rest and immediately replaced it. He did the same with the first book on the shelf above and continued; third shelf - second book, fourth shelf - third book, fifth shelf - fifth book, last shelf - eighth book, which he left halfway dislodged. To the students surprise the bookshelf rotated, as if hinged in the middle, perpendicular to the wall and leaving an opening sufficient so that a generously proportioned adult might pass. Inside was a passageway, a rough-hewed rock wall with a cobblestone floor. Dimly burning gas lanterns periodically dotted the wall that, as far as the eye could see, appeared to be absent of entrance or exit save the bookshelf that rotated closed upon the foursomes entry as Blanchard pushed the eighth book back into place. The students silently followed the old professor, too bewildered to ask questions. Blanchard’s brisk stride belied his years. There was a perceptible down slope to the floor and the group seemed to gather speed as they traversed the straight path to who-knows-where. After what seemed like an eternity, but in actuality was more likely ten to fifteen minutes, their expedition abruptly ended at what for all intents and purposes appeared a dead end. On the wall barricading their further progress was engraved dozens of symbols; Arabic, Ancient Greek, Hebrew, Babylonian, and mathematical, among others unidentifiable to myself. In succession Blanchard rapped alternately on the Greek and Hebrew letters ε – 7 – ε – 7]. Epsilon – Daleth – Epsilon – Nun.

The students turned to each other, silently mouthing the letters. Then in unison chorused,

“EDEN!”

The wall obeyed the apparent command to open, revealing a well-lit but totally chaotic laboratory. Vials were overturned and broken. Burners, pipettes, test tubes, and notes littered the floor, which was covered with shards of a glassy-like gelatin substance. A faint scent of smoke perfumed the dank air, an odd odor similar to yet distinct from spent gunpowder. Judging from a slight breeze that wafted through, the lab was apparently ventilated but there were no windows or doors. The only exit was the entrance. Cabinets lined three walls; most of their doors ajar and only a sparse collection of canisters labeled *Iodine* populated their shelves. Obviously, the majority of what had filled them now lay

on the floor, either empty, destroyed, or pummeled into a state of extreme distress. A generator whirred away loudly in one corner, a maze of wires and instrumentation issuing in all directions. Books on chemistry, physics, mathematics, and biology randomly decorated every quadrant.

The four entered and as the students surveyed the ruin, trying to appraise their circumstance and surroundings, Blanchard strode purposefully to a table covered with a burlap sheet at the far end of the room. He righted four overturned stools and beckoned his students to join him for a seat.

“Welcome to Eden my young friends. This, my laboratory for almost as long as your collective years, has not always resembled Jerusalem after the Romans. Other than my late beloved Sylvia - wife, lover, friend, and colleague for two-thirds a century, you are the only human beings to set foot on these premises. Now, I said I had much to tell you, but first I wish to hear what you have learned and why you have come to me in such desperation at this hour. Granted, looking upon myself it is indeed ironic I should term your current state as desperate, but please, enlighten me.”

Having taken their seats and still somewhat breathless and disoriented from their flight down the corridor and subsequent landing in Eden, Ian, Cayley, and Julia paused, looking at one another to see who should speak. Ian took up the task.

“Dr. Blanchard, please excuse us for interrupting you at an obviously inconvenient moment, but we think we understand your assignment. An innocuous comment in the student newspaper sent us in haste to your office.”

“Go on.”

“We believe, please don’t mock us, that you are speaking literally when referring to a Tessellate and her state. After two weeks of poring over your writings and referencing your footnoted works, in particular the mathematical sources, though most is beyond our grasp we get the gist. Our conclusion, in summary, is this: you have discovered how to create an elementary form of life, but some sort of problem has arisen. It must needs be a problem that you are ill equipped to manage on your own, otherwise you would simply solve the predicament yourself. Mention of your deceased wife, and please accept our sympathy, confirms our assumption that you are a solitary researcher and stand in need of assistance.”

“Fascinating. Do go on Mr. Wainwright.” Blanchard smiled wryly.

“We can’t decide if your problem is ethical in nature or if you simply need more hands and feet to direct your project properly, but we have come to help.”

“And why should I include uninitiated students, even less than novices, in as lofty a quest as you describe?” His smile flattened, turning a very serious visage on the group.

“Because Professor”, Julia interjected, “the only person or persons you could entrust with such knowledge would be someone who shared your passion. You knew that anyone coming to the conclusions that we have, regardless of age or lack of understanding, could not do so unaffected. We had sooner die than betray your confidence. This must sound ridiculous after only a few days of research but, as Ian stated, we offer ourselves in your service.”

“And what of the innocuous quote that sent you in such alacrity and determination to my office?”

“Dr. Blanchard, I thought you a nutter, on the brink of the sanatorium.”

He raised one brow as he turned to hear Cayley finish her thought.

“I’m so sorry sir, but I don’t often mince words.”

Blanchard smiled, as he knew this to be one of Ms. Cayley’s defining characteristics.

“But I know now, or at least I feel strongly, that you may be the sanest man under which I’ve had the privilege to study. When asked by the editor of the student newspaper how you planned to occupy yourself away from the classroom as you are now approaching retirement, you replied your only wish was to spend more time with your family. In particular, you cherished the notion of nurturing the newborns. The editor then asked if you would name a favorite. Your reply – Tess! We could wait no longer and came immediately. Accept my apology for my earlier insolence of thought and do make us of service.”

The old professor sighed and leaned back against the table, eyes glistening. It was difficult to tell if from a tear or delight, perhaps both, but he had heard enough. He stood, placed a hand on Cayley’s shoulder and surveyed the trio.

“Brilliant! Simply brilliant. You are right on all accounts. I am moved by your acumen and your faith. I gladly accept your offer.”

Ian, Cayley, and Julia beamed.

“Now allow me to follow through with my promise of illumination.”

Blanchard assumed the familiar stance of the lecturing professor to the eager pupils, comfortable roles for all.

“I will not weary you with unnecessary detail but these are the essentials. Crystalline structures, molecules that have regular geometric internal arrangement, provide us with some of the most stable rigid chemical formations known. All crystal substances grow to produce the same angles between their faces. Some regular crystals can be sheered to lay in a plane and in essence produce a two-dimensional field of replicas. Even the fields that

are not replicas of the same polygon can be combined with other polygons to produce a tessellation.”

“A figure or group of figures that cover their plane of construction with no overlaps or gaps.” Julia grinned with satisfaction. Prof winked and continued.

“What I have discovered is a way to produce regular, carbon-based, tessellating crystalline structures. By following a complex succession of mathematically precise steps, I can excite the structures, create enormous energy and, friends, bring the tessellation to life.”

“God’s Equation Professor?”

Blanchard smiled impishly. “More accurately, equations Ms. Cayley. The usage of the singular in my essays was for effect; a bit of literary license so-to-speak.”

Ian, Julia, and Cayley were on the edge of their stools. Even though they had anticipated as much, it was riveting to hear the professor confirm to their hearing the most important discovery in the history of mankind. To animate the inanimate was to become a god. Could it truly be?

“Let me show you.”

At that, Professor Blanchard removed the coarse sheet from the table, revealing three square trays of approximately one meter square apiece. On each was what appeared to be a mosaic of geometric shapes. One was solely equilateral triangles of three centimeters, another solely regular hexagons of three centimeters, but the third was a combination of three regular figures, all of three centimeters, one being central, a decagon. Alternately on each of its ten sides were a regular square and a regular hexagon. Any decagon was bordered by five squares and five hexagons. Two each of the other figures bordered each square, and each hexagon evenly shared its borders with decagons and squares. It was beautiful but all three appeared static.

Julia asked, “How can you tell these are alive professor?”

“Oh, alive they are not. Not yet, Ms. Spenser. What you see are three pre-life forms. The carbon-crystalline structure is in place, awaiting the enlivening process.”

“Then let us see it professor. I can’t bear waiting a moment longer.” Ian was beside himself and Cayley, for perhaps the first time in her life, could not speak.

“But therein lays my problem my new found aides. And this is where you come in; fresh insight, youthful perspective, physical strength to bolster those attributes waning in myself.”

“I don’t understand Dr. Blanchard.”

“This is the situation Ms. Spenser. When brought to life the Tessellates remain docile and stable but exhibit signs of life immediately. They give off heat. There is obvious respiration and they require food to live.”

“Food?”

“Yes. Food. That is part of the mathematical aspect. Depending on their angle and side length, each tessellate must have a specific diet of oxygen enhanced iodine to survive. If not, they will expire within minutes. As you can see from the empty vials and disarray of my inventory, I have exhausted almost my entire supply of iodine. That, however, is not the foremost problem. The longer the Tessellates live, the more signs of consciousness develop. In short, they are aware of where they are and unmistakable signs of inquiry appear. They attempt to move, often independent of one another, but being attached as they are, it is a dangerous attempt at locomotion. If one moves so that a shared side divides with its neighbor, and it always divides, the entire tessellation attempts to adjust to fill the unnatural gap and consequent equivalent overlap. By unalterable rule of their creation, they can tolerate no such aberrations. If the gap is not filled, the overlap eliminated, and the regularity of the planes occupants restored immediately, the entirety of their tessellworld explodes with such force and violence that it has nearly proved my demise on numerous occasions. Tessellands of only one shape seem to be the most successful at these attempts at eliminating aberrations. One Tessellate rotates, essentially instantaneously, around a common angle perfectly on top of the Tessellate with which it shares the angle of rotation. The melding of the two seems to be of mutual consent and I can only surmise that it is agreed upon before hand. The most curious aspect of the rotations is that they seem to be ever and always directed toward my person. It is this persistence that eventually, and to this point invariably, concludes in termination.”

“Maybe they are just moving toward a heat source Professor.”

“I thought just the same, Ms. Cayley and constructed numerous experiments to test that hypothesis. In every case, without fail the movement was still in my direction. When I observed from what once was a one way viewing cove...”

Blanchard pointed to a small chamber obviously missing a window.

“...the movement persisted. The chamber was completely heat and sound insulated, so the only logical conclusion: I am in some way key to their desire. Most distressing is that at the moment of annihilation, when movement has produced an irreconcilable gap-overlap, I hear a distinct cry of pain and remorse.”

“Your ‘Creator’s Dilemma’ Professor?”

“Exactly, Ms. Spenser. Exactly. I must maintain constant vigil and uniform circular progress around their tesselland, insuring their movement is altered enough to prevent the inevitable self-destruction. This has caused me no trivial anxiety, my fine pupils, and has

been my occupation these last few months. Still, sadly I fail at each attempt. My affection for these beings has grown and I feel compelled to supply their needs, secure fulfillment of their ambition, save them from their pain. But I know not how, save never again to create. I can not abide their death pangs.”

Blanchard’s expression changed to resigned disconsolation as he slumped back to his stool. No words were spoken but Cayley arose, walked to the aging teacher and empathetically embraced him. The whole group, sobered by the professor’s comments, remained in silence for several moments, each contemplating the quandary. Julia stood.

“Professor, I think they seek to be nearer their creator. Like us, their creator exists in a different dimension. They sense its presence and gravitate to it despite self-annihilation, yet irresistibly drawn. It is their subconscious and conscious striving in one.”

“Dear Heart, if you are correct then the pain is even more acute.”

Releasing Blanchard, Cayley suggested,

“Sir, we need only change their dimension. If instead of a plane of construction, you created a space of construction, the tessellates might be more able to satisfy their urge, fulfill their desire, meet their creator.”

“That is a grand notion, my other dear heart, but achieving a transformation from two to three dimensions is most probably a continuous metamorphosis, and one I am not prepared to undertake. I am neither enough a mathematician nor scientist for the task; at any rate, not yet, and I doubt this hoary head sees enough years to conquer the charge. This will necessarily be the achievement of younger minds.”

Ian leapt to his feet and shouted,

“Homolosine!”

“What?” the others said in unison.

“The homolosine projection. To display in two dimensions a three dimensional sphere without the lateral polar area distortions of Mercator’s projection or the polar shape distortions of the sinusoidal projection, the homolosine ‘orange peel’ map was created. Why not reverse the process?”

After a brief pause, Blanchard bolted upright.

“A younger mind indeed! Yes Ian. Yes.”

The atmosphere became alive with hope.

“Amazingly brilliant and amazingly simple. I’m humbled by your insight and my oversight of the obvious. Instead of a square plane of construction, if a tesselland is generated on a homolosine plane... It may just work, my friends. It ...may...just...work.”

The professor, closely followed by the students, went to the table of trays. He scribbled a few calculations on some of the scattered notes lying nearby.

“Yes. That should do it.”

The students watched in silent awe and anticipation. Retrieving the tray of equilateral carbon-crystals, Blanchard took a scalpel and carefully cut a homolosine plane into its soft waxy contents. He removed the excess from outside its periphery and quickly but cautiously, began connecting cables from the still whirring generator to nodes on the sides of the tray. He then went to the panel board from which the cables emanated, pulled a notebook from his tattered overcoat, obviously checking, rechecking, and verifying calibrations, then began setting a series of dials, gauges, and switches. When satisfied that all was in proper order, he turned to the students.

“Ian, Julia, Cayley. You best remove yourselves outside this chamber in case something goes terribly awry.”

“No!” “No!” “No Professor” They shot back in response.

Realizing the futility of asking again, Blanchard suggested they crouch in the viewing chamber. This was acceptable and the three hunkered in impatient expectancy and hope. The professor paced back to the panel, then to the tray, then back to the panel, rechecked his calculations, took a deep breath, raised his hand to a switch, looked to Ian, Julia, and Cayley with an expression both eager and fearful, then pulled the lever.

A gentle hum grew to a low bass roar, the tray vibrated, glowed a beautiful amber tone that soon changed to violet. Blanchard, having pulled his timepiece from his pants pocket, counted out loud, though he could not be heard over the generator, then flipped the switch off. Instantly the roar subsided and the tray returned to normal. The students, unable to contain themselves sprang from the viewing chamber.

“Stop!” Blanchard commanded while rushing toward them.

“That wasn’t the dangerous part. That begins now. Back in chamber quickly.”

As he was addressing the students, the contents of the tray began to pulsate and the lines of the triangles began to blur. Blanchard darted to a shelf and clutched one of the still intact bottles of iodine. From a drawer under the table he lifted a black bottle, removed its cap and exacted three tablets, added them to the iodine and using what appeared to be a soup ladle, spooned one scoop of the mixture over the tray. The triangles stabilized and their lines became distinct and clear. The professor stood over the tray, peering down, a

study of rapt concentration. With the tray fixed, its h-plane of triangles began perceptibly to move toward him. Realizing his mistake, he began circling the table at a normal walking pace. The h-triangles steadied then, amazingly, began to rise in the center. Blanchard paused in astonishment. It was working. The plane and its occupant triangles was transforming, but suddenly, just as the h-plane was folding up on itself, a bloodcurdling screech and heart-rending moan issued from the transforming mass. A brilliant flash and deafening boom announced the self-destruction of the brief homolosine tessellworld. Shrapnel flew in all directions, knocking Blanchard off his feet and peppering Ian and Julia. Cayley ducked below the viewing ledge just in time. A distinct odor of spent gunpowder filled the room. The professor, bruised but not broken, managed to regain his feet and stood staring, tears in his eyes over the debris. Julia, unhurt, was also in tears.

“Oh, Professor. I heard it. They cried. They were in anguish and cried. Oh, dear God.”

Ian, pieces of the h-triangles scattered over his torso, held Cayley.

“No more! Never again!”

Blanchard was distraught. He stalked to the entrance of the lab but Ian cut him off.

“Sir. We have to try again.”

The professor turned to avoid him but Ian grabbed him by the shoulders.

“I think I know what went wrong sir.”

“Ian, I will never subject myself or those poor beings to that fate again on the strength of another hypothesis. Even though there appeared a chance it might succeed, the end precludes the means. No!”

“But sir, please listen. You are the reason.”

“Brilliant, young man. You have just stated the redundantly obvious. Is this your solution? Ridicule, humiliate, and indict me? Fine solution indeed! I will now take my leave and unless you know the exiting combinations for the lab and passageway, I suggest you and these young ladies join me.”

“No. Sir. No. That’s not what I intend. I mean I don’t intend to ridicule or implicate. Please, just listen and if you still insist on abandoning this quest, we will join your resignation.”

Blanchard sighed, slumped his shoulders, and turned back to Ian.

“You have precious few sands in the hourglass. Proceed.”

“Sir, you yourself saw that it was working. The transformation was taking place but their effort toward you was skewed and complicated by the attempt to transform. If we position you better, I’m convinced we will succeed. Please try one more time. Just one more time.”

Blanchard scratched his head, shuffled his feet and looked up at Ian. Julia and Cayley motionlessly waited for his response.

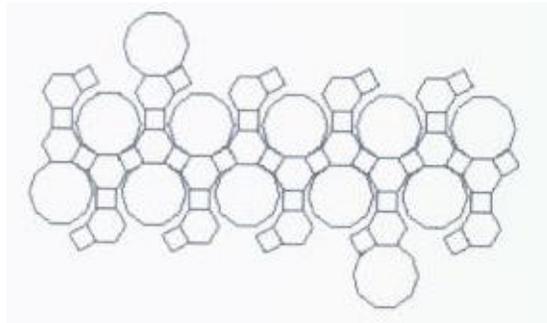
“Hmmm. You may be right yet again young man. Skewed their effort. Symmetry. Concentric and equal applications of energy.”

He surveyed the room, floor to ceiling.

“Help me slide the table under this cable.”

The four, with much effort pushed, pulled, and tugged the table until it was directly underneath a heavy cable, obviously used to hoist and hold weighty items.

“The hexagon tray is useless. The blast destroyed most of the crystals but the other tray is unscathed. It appears the blast fortuitously propelled the burlap wrap over its contents. Help me clear the table and set up the apparatus.”



They proceeded with resolve and soon everything was in place just as before.

“All that remains is for the switch to be thrown and the solution to be added once the tessellates are enlivened. Cayley, would you do the life-giving honors and Julia, you be prepared with one ladle of solution. Ian, I will need your muscle. Unloose the cable from its hook on the wall.”

Cayley took her place by the control panel. Julia had the solution poured and ready. Ian loosened the cable and Professor Blanchard straddled the tray, which lay directly below the line. Blanchard then secured the rope around his waist with a knot that would make a sailor in Her Majesty’s Navy proud.

“Now, with all your masculine fortitude, hoist me above the table Ian.”

Ian, with surprisingly little effort, raised the professor several meters above the table and tray, the homolosine tesselland directly below.

“Listen carefully. Should things go badly again, you must save yourselves. This is a more complex carbon-crystal than the previous one. It contains much more energy. The blast will surely prove fatal if you are not behind the viewing chamber’s walls. Even if destruction is imminent, I will have no opportunity, nor desire, for escape. I choose now to perish with my...”,

turning and looking appreciatively, even lovingly at Ian, Julia, and Cayley,

“...our creation. Ian, immediately upon Ms. Spenser’s application of the solution, lower me within one meter of the tray, secure the rope to its hook and join your friends in the viewing chamber. Stay completely below the aperture. Mind you, that still may not save you. Do you understand? Are you willing the risk?”

They gave a definitive and confirming nod, giving each other a supportive wink and thumbs up.

“OK Ms. Anne Margaret Cayley. Pull the switch”

Cayley complied. The mechanism sprang to action; hum, bass roar, amber then violet glow. Blanchard, watching his timepiece, gave her the signal to terminate power. Again, she complied. Julia moved efficiently to the tray and spooned the sustaining nourishment over the tessellates. She and Cayley retreated to the chamber as Ian lowered Blanchard to the prescribed height. As he secured the cable to its hook, the homolosine tessellates began their dimensional transformation. The professor coiled himself into as tight a ball as possible. The tesselland rose in the center first, the edges following. All looked perfect then the mass began vibrating violently.

“Ian, save yourself.”

Ian, instead of moving to the chamber, bolted toward the table.

“No Ian!”

A brilliant flash of blue light and intense, almost unbearable heat issued from the rising homolosine tesselland and then, quiet. No explosion. Ian lay prone but conscious on the floor. Cayley and Julia were slowly emerging from the chamber, their mouths open and eyes wide in disbelief. Floating into Professor Blanchard’s grip was the most astoundingly beautiful thing any of them had had the pleasure to behold.

It was a pulsating, multicolored sphere. In actuality it was not exactly a sphere. As Ian got to his feet, staring at this new wonder, Cayley, her voice cracking, spoke.

“Professor. It’s a great rhombicosidodecahedron.”

“That it is my dear. That it is!”

Blanchard beamed broadly behind his charred beard and blackened face as he embraced the newborn. It hummed contentedly in his arms as he hung, still suspended from the ceiling and gently swaying from side to side.



“Ian. Good Show. You must have realized that the extra energy expended in transformation necessarily demanded more initial oxi-iodine than normal. Jolly good show dosing the tesselland world as you did with Julia’s remaining solution. Saved the lot of us. Good show all!”

Predictably, I awoke the next morning in an exaggerated state of excitement and energy. Indeed, my wife was suspicious and had she not had such a fitful nights rest due to my enchantment, noting my presence as I oft awakened her, would have thought me surreptitiously carousing with dubious company. Not able to contain my enthusiasm, I immediately retold the encounter. Suggesting I rethink the volume of brandy in my nightly toddies and glancing instructively toward the rubbish bin, she rose from breakfast. I did my duty. The following days, I went about business as usual but unable to escape the notion my vicarious adventure was not over. If my hypothesis was correct, I could expect one more visitation. In fact, as I settled for my traditional midweek afternoon nap, which happened to mark two fortnights post-first reverie, I prepared to see my friends. I was rewarded.

“Students, you have done excellent work. Some have risen above the others in accomplishment, but this will always be the case. None, save one, have shamed themselves.”

A sheepish young man shrank in his seat slightly as Professor Blanchard gave him a disdainful glance. Turning back to the class, the professor displayed something in his palm. He concluded his lecture.

“This is a small wooden model of the eleventh Archimedean solid – a great rhombicosidodecahedron. Is it not beautiful in a somewhat esoteric way?”

The students agreed.

“It is commonly believed that great ideas – philosophical, mathematical and otherwise – express higher realities. We all strive for a higher plane of existence, to be nearer to that which is nobler, more decent, more righteous, more gracious than ourselves. Never cease to strive for that which seems beyond the limits of your reach. I will dismiss you with a quote from the psalmist.

‘As the hart panteth after the water brooks, so panteth my soul after thee, O God.’

Dismissed.”

Slowly, lazily, the students gathered their bags and wraps, leaving randomly as students are prone to do. Three tarried.

The earth has once orbited its sun since that last lecture by Professor Blanchard. I have missed him, Ian, Julia, and Cayley as I would miss my own family, but they live as vividly in my memory as if they occupied the seats next to me on my daily commute to work. My wife is just now beckoning.

“Dear. You have a parcel.”

“Whoever from?”

“Odd. The return is marked only *‘Department of Philosophy’*”

“I’ll have that please.”

“Judging from its size and shape, I’d say some chap has boxed and mailed you an apple. Well, my goodness. What is that?”

“Oh, my. Oh, oh, my.”

“What an unusually peculiar but beautiful piece. But dear, what is it?”

“Oh, my.”

“What is the bother with you?”

“I’m sorry dear. Uh, I d...d...do believe it is a great rhombicosidodecahedron.”

“A WHAT?”

“A wooden model of the eleventh Archimedean solid.”

“You mean the what’s-it you told me about months ago when you were having those strange dreams?”

“The very same.”

“And what is that engraved on one of the large surfaces?”

“Psalm 42:1”

“Psalm 42:1? I wonder what it says.”

“I think I know. No dear, I’m quite sure I know!”

THE END

For the interested reader:

- To view the Archimedean Solids, go to

<http://www.mathworld.wolfram.com>

and follow these links

- **Geometry**
- **Solid Geometry**
- **Polyhedra**
- **Archimedean Solids**

This site is a wealth of all kinds of mathematical information.

- To make a tessellation, go to

http://arcytech.org/java/patterns/patterns_j.shtml

- For more on tessellations, go to

<http://mathforum.com/sum95/suzanne/whattess.html>

- To view the amazing tessellation art of M.C. Escher, go to

<http://www.mcescher.com/>

and follow these links

- **Gallery**
- **Symmetry; most of M.C. Escher’s Symmetry Drawings**

