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Innumeracy and Its Perils, Numeracy and Its Promises

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Introduction

It is socially unacceptable to be illiterate, but not to be innumerate (lacking in basic mathematical ability). Taking examples from Paulos (1997), Steen (1997), and others, I will discuss the importance of numeracy, how it can make one more conscious of laws of our society (e.g., regarding speed limits and speeding), and how it allows one a more critical view of dubious claims (such as stocks prediction scams) and mandatory medical tests (such as AIDS test). In addition, being numerate allows one to compute or estimate very quickly in everyday contexts (such as in leaving a tip or computing pulse rate after an aerobics workout). In a lighter vein, I also discuss how numeracy allows one to understand how some "magic" tricks (such as some number puzzles) work. In short, I will show some perils associated with innumeracy, and some benefits of being numerate.

Speed limits

Most of us believe that a speed limit of about 30 mph is extremely slow, and possibly unreasonable. But 30 mph is equivalent to 44 fps. Given that 44 feet is the length of a volleyball or badminton court, 44 fps or 30 mph is the same as traversing the length--44 feet--of such a court, in one second. Assume that one is traveling at 30 mph, and one sees a pedestrian crossing, about 40 feet ahead. If one takes 0.4 seconds to react, this means one has already traveled 0.4 x 44 feet = 17.6 feet in that time. Assume, too, that one's car brakes are good, and that it takes 0.6 seconds for the car to come to a complete stop, after the brakes are applied. Then, one would have traveled 0.6 x 44 feet = 26.4 feet before coming to a complete stop. Hence, once would have traveled a total of 44 feet in the one second it takes to react and make the car come to a complete stop. Since the pedestrian was about 40 feet away, the pedestrian would have been struck, (assuming one did not swerve to avoid the accident—which might have caused some other accident!). All this, even though one was traveling at a "slow" 30 mph! If one takes into account that one seldom travels slower than 30 mph, and that driving conditions are not ideal (for example, the road might have been slick due to a recent downpour), and one's reaction time and the brakes are not very good, it makes sense to be aware of the potential danger one is exposing oneself to, when one travels at any speed above the posted speed limit. Note, too, that at 60 mph, the distance traveled is 88 feet per second—a sobering thought indeed!

It is a common rule of thumb that stopping distance is a car length between cars for every 10 mph one travels; at 30 mph, then, we should maintain at least 3 car lengths, at 60 mph, at least 6 car lengths, an so on. This rule actually underestimates the recommended stopping distance that is taught in many "safe driving" courses (the "2 second" rule), just as it underestimates the stopping distance that is calculated by using the relevant quadratic function relating stopping distance to the car's speed, the driver's reaction time, etc. For more information on stopping distances, the reader may access the following URLs: http://www.co.honolulu.hi.us/hr/safefollowingdistance.pdf, and http://www.unb.ca/web/transpo/mynet/mty107.htm.

Teachers will find the following site related to stopping distance, especially useful:

http://www.phschool.com/atschool/Mathematics/Algebra/Student_Area/ALG_SC7_ACT1.html.

At this site, teachers can get good ideas and activities that link algebra to stopping distance.

Stock market predictions (modified from Paulos, 1997, p. 43)

Assume that a non-scrupulous stock market analyst mails out 32,000 letters where she predicts in 16,000 of the letters that a certain stock, X, will go up, and she predicts in the other 16, 000 letters that the stock will fall. Assume, for the sake of simplicity, that the stock can only go up or down, and that the stock X in fact went up. Now, she mails out 16, 000 letters to those 16, 000 who received a correct first prediction, this time predicting that a different stock Y will go up (in 8, 000 of the letters), and down (in the remaining 8, 000 letters). Assume the stock Y went up this time, too. Now, she does the same thing with the next 8, 000 letters, where half of the group receives the prediction that the stock Z will go up, and the other half, the opposite prediction. Assume the prediction this third time around is correct for 4, 000 of the group. Effectively, then, we have now a total of 3 consecutive, correct predictions, for 4, 000 people. Let's say that all these predictions were given free of charge. Now, the analyst sends out a letter stating that as she had correctly predicted the movement of the stock 3 consecutive times, without any cost to the recipients, she is prepared to mail out the next prediction to anyone who pays her $100 for the next prediction. And, to make the offer more attractive, she states that she is prepared to give an "iron-clad money back guarantee," to anyone who gets a wrong prediction this 4^{th} time around. Since \$100 seems a pretty good deal for getting an accurate prediction that might net thousands, let's say that she gets 4, 000 x \$100 = \$400, 000, from the 4, 000 people who agreed. This time around, then, out of the 4, 000 letters, only 2, 000 had the correct predictions, so she has to return 2, 000 x \$100 = \$200, 000, to those who received the wrong prediction. But, she can now laugh all the way to the bank, being \$200, 000 richer !

AIDS Test (modified from Paulos, 1997, p. 89-90)

Assume an AIDS test for positivity is 99% accurate. This means that, 1% of the time it gives "false positives," that is, it gives a positive result of AIDS 1 % of the time, even though the persons that tested positive for AIDS does not actually have AIDS. Assume also that ½ % of the population have AIDS. So, if we take a random sample of 20, 000 people, about 100 of them (½ % of 20, 000) would have AIDS, and 19, 900 would be free of the disease. But if we administer the AIDS test that is 99% accurate, we will find 99% of the 100 who have AIDS – 99 persons--testing positive. Also, we will find 1% of 19, 900, who do not have AIDS –199 persons—testing as "false positives." That is, 199 of people tested will test positive for AIDS in spite of not having AIDS. Therefore, altogether 99 + 199 = 298 people test positive for AIDS. Hence, the probability of a person having AIDS, given that they test positive is 99/298 ≈ 33 %.

Compare this with the probability of a person testing positive for AIDS, given that the person in fact has the disease. In other words, suppose

A = the event a person has AIDS, and

Pos = the event that the person tests positive,

with

P[A / Pos] = the probability the person has AIDS, given they test positive, and

P[Pos / A] = the probability the person tests positive, given they have AIDS .

With this vocabulary , P[A / Pos] and P[Pos / A] are conditional probabilities, with

P[Pos / A] = .99, and P[A / Pos] = .33, and P[Pos / A] $\neq$ P[A / Pos].

Note, too, that hardly any test is 99% accurate in testing for positives. Most tests predict positives accurately only 75% to 95% of the time. The corresponding conditional probability of P[A / Pos] ranges from 1.5% to 9%. The lesson here is that, not only does one not have to worry unduly if one tests positive for a disease, given such a test is between 75% and 95% accurate, but that it is unnecessarily costly, wasteful, inaccurate, and stressful to have government-mandated AIDS (or other) tests for every person in a country. In short, a little understanding of conditional probability can go a long way, both in easing anxiety in an individual and in constructing useful medical policy.

Much the same message, but related to the statistics about his own cancer, is given by Stephen Jay Gould, in his article, “The Median Isn't the Message,” that is reproduced at the site http://www.cancerguide.org/median_not_msg.html. For more on numeracy related to statistics, the reader is urged to read the following:
Best (2001; http://www.statlit.org/PDF/2002BestASA.pdf), and Duff (1954).

Calculating a Tip Mentally

It is customary to tip at restaurants, and a 15% tip is generally considered appropriate. Using the fact that 15% = 10% + 5% = 10% + (½ x 10%) a tip can be computed very quickly. For example, if a bill comes to $42.00, take 10% of this amount—$4.20, and add half again as much—$4.20 + $2.10 = $5.30.

Computing Pulse Rate After an Aerobics Workout

Many health-conscious individuals join health clubs, and do regular aerobic workouts. After a workout, the aerobics instructor usually suggests computing the pulse rate by counting the number of heartbeats in a 10 second interval, and then multiplying that by 6, to get the pulse rate for one minute (as 10sec x 6 = 60 sec = 1 minute). For instance, if the number of heartbeats is 28 in 10 seconds, then we have 28 x 6 = 168 beats in one minute. Most of us are not able to compute 28 x 6 accurately and quickly while jogging! But, if we find the number of heartbeats in 6 seconds, and then multiply that number by 10, the computation is much easier. Say, then, that the number of heartbeats for 6 seconds is 17. Then multiplying 17 by 10 gives 170, the pulse rate for 1 minute.

Number Puzzles

The general public is usually amazed at the seemingly marvelous feats performed by those who use number puzzles to predict a certain number or event. Such puzzles can be heard on talk shows by talk show hosts, or comedians when the host wants to stun the public to grab their attention. One might hear gasps from the audience. What the public seldom realizes is that most of these number puzzles have a mathematical basis that can be explained by using deductive logic or by using algebra.

Let's discuss an old number trick that was very popular in magic shows at one time. The directions are as follows (the numbers in parentheses are examples of a set of possible responses):

Step 1: Write down the year you were born. (1960)

Step 2: Write down a year when a memorable event in your life took place. (1990)

Step 3: Write down your age as of the last day of this year. (2003 – 1960 = 43)

Step 4: Write down the number of years that elapsed since the memorable event (in Step 2) took place. (2003 – 1990 = 13)

Step 5: Add the 4 numbers obtained from Steps 1 through 4. (1960 + 1990 + 43 + 13 = 4006 = 2 x 2003)

Prediction: All of you have the number 4006 (if the trick has been carried out in the year 2003).

The prediction is amazing, but simply explained.

Explanation: Look at the numbers added in Step 5:

1960 + 1990 + 43 + 13 = 1960 + 1990 + (2003 –1960) + (2003 – 1990)

= 1960 + 1990 + 2003 –1960 + 2003 – 1990 (removing parentheses)

= 2003 + 2003

= 4006 = 2 (2003)

Note how we simply deleted 1960 and 1990. (In fact, most number puzzles undo what has been done, so as to "force" a certain result, as can be easily proved by using deductive logic or by using algebra.)

Conclusion

We have tried to offer some examples where numeracy leads to useful and efficient applications of math to daily life. Mathematicians may find these results completely natural; we are probably quite familiar with these types of findings. But many individuals who resisted their lower school mathematics courses saying, "we will never use this," can be reminded with these examples how much numeracy improves our understanding of the world. As teachers, we can suggest these types of examples almost daily to our classes, with very little expense of time. Students learning Calculus would like to say, "we are never going to use this stuff." But when simple examples such as these are narrated at the beginning of the class period, say, students gradually come to see that practical mathematics is usually there if you know where to look. Hopefully, the students will then gain more appreciation for the mathematics they are learning in today's lesson.

(Note: A good resource for teachers wishing to use numeracy-related issues arising out of newspaper articles is the site http://ink.news.com.au/mercury/mathguys/mercindx.htm, which contains full-text newspaper articles from Australian newspapers, and ideas on how to use them in the classroom.)

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